1. What is the most important assumption we make when we linearize an equation?

2. Explain the difference between a zonal average and a meridional average.

3. What is the formula for the speed of a Rossby wave when \( \bar{u} = 0 \) and the north-south wavenumber \( l=0 \)?

4. What do we mean by a dispersive wave?

5. Give typical mid-latitude values for \( f, \zeta, \) and \( \eta \).
1. Consider the impact of friction on the phase speed of Rossby waves. We can still examine the barotropic vorticity equation but with friction \( \mathbf{F} \) included in the form: \( \mathbf{F} = -\mu \mathbf{V} \), where \( \mu \) is a positive constant, and \( \mathbf{V} \) is the horizontal velocity vector.

The linearized barotropic vorticity equation in this case is:

\[
\frac{\partial}{\partial t} (\nabla^2 \psi') + u \frac{\partial}{\partial x} (\nabla^2 \psi') + \beta \frac{\partial \psi'}{\partial x} + \mu \nabla^2 \psi' = 0,
\]

where all notation is as in class.

(a) Assuming \( \psi' = A \exp[\text{i}(k(x-ct))] \), find an expression for the phase speed \( c \) of Rossby waves influenced by friction.

2. For the pair of linearized equations below, assume a wave-like solution of the form \( (u', v') = (A, B) e^{i(k(x-ct))} \), and thus find the expression for the phase speed \( c \). Assume below that \( \bar{u} \) and \( f \) are constants.

\[
\begin{align*}
\frac{\partial u'}{\partial t} + u \frac{\partial u'}{\partial x} - f v' &= 0, \\
\frac{\partial v'}{\partial t} + u \frac{\partial v'}{\partial x} + f u' &= 0.
\end{align*}
\]

3. When the mean zonal wind \( \bar{u} \) is a function of latitude (i.e., \( \bar{u} = \bar{u}(y) \)), the linearized barotropic vorticity equation for frictionless flow becomes:

\[
\frac{\partial}{\partial t} (\nabla^2 \psi') + \bar{u} \frac{\partial}{\partial x} (\nabla^2 \psi') - (\bar{u}_{yy} - \beta) \frac{\partial \psi'}{\partial x} = 0, \tag{I}
\]

where \( \psi' \) is the disturbance streamfunction, and \( \bar{u}_{yy} = d^2 \bar{u} / dy^2 \).

Suppose we assume a solution to (I) of the form:

\[
\psi'(x,y,t) = \Psi(y) \exp[\text{i}(k(x-ct))]. \tag{II}
\]

Find the ODE which must be satisfied by the amplitude function \( \Psi(y) \) if (II) is to be a valid solution of (I). Do not attempt to solve this ODE.